

# A note on bigravity and dark matter

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We show that a class of bi-gravity theories contain solutions describing dark matter. A particular member of this class is also shown to be equivalent to the Eddington-Born-Infeld gravity, recently proposed as a candidate for dark matter. Bigravity theories also have cosmological de Sitter backgrounds and we find solutions interpolating between matter and acceleration eras.

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Observations show that most of the energy density in the universe is in the form of dark matter and dark energy [1, 2, 3, 4, 5]. It is therefore of importance to have a simple and natural candidate for these components.

Motivated by Yang-Mills theory, multi-graviton actions are attractive extensions of general relativity. Consider several metrics  $g_{\mu\nu}^a$ ,  $a = 1..n$ . The physical properties and geometric interpretation of such a theory present great challenges. It is known that the full  $(\text{diff})^n$  symmetry cannot be preserved by consistent interactions [6]. The most general action preserving the full symmetry group is a sum of decoupled Einstein-Hilbert terms for each metric.

However, interesting theories can be built by breaking the  $(\text{diff})^n$  symmetry down to the diagonal subgroup. For  $n = 2$ , a particular “bi-gravity” theory with metrics  $\{g_{\mu\nu}, q_{\mu\nu}\}$  and action

$$I = \frac{1}{16\pi G} \int \left\{ \sqrt{-g}(R - 2\Lambda) + \sqrt{-q}(K - 2\lambda) + \frac{1}{\ell^2} \sqrt{-q} \left[ -q^{\alpha\beta} g_{\alpha\beta} + \kappa \left( (q^{\alpha\beta} g_{\alpha\beta})^2 - q^{\alpha\beta} g_{\beta\gamma} q^{\gamma\delta} g_{\delta\alpha} \right) \right] \right\}, \quad (1)$$

has been extensively studied [7, 8, 9, 10, 11, 12, 13, 14, 15]. Here  $\Lambda$  and  $\lambda$  are cosmological constants for each sector of the theory.  $q^{\mu\nu}$  and  $g^{\mu\nu}$  are the inverses of  $q_{\mu\nu}$  and  $g_{\mu\nu}$  respectively.  $K$  is the Ricci scalar for the metric  $q_{\mu\nu}$ .  $\kappa$  is a dimensionless coupling. A class of bi-measure theories have been considered in [16] and references therein.

The interaction term above was first proposed in Ref. [7] (see note [36]). A key feature of this interaction term is that it gives rise to Fierz-Pauli mass terms[17] for the spin-2 fields. The above is not the most general mixing term that satisfies this condition. In particular, the density  $\sqrt{-q}$  could be replaced by  $(-q)^u (-g)^{1/2-u}$  for any real  $u$ . However, for this theory to give rise to a dark matter dominated era, we find that, under the assumptions described below,  $u = 1/2$  is required.

In this short note we point out the following properties of (1). First we prove that for  $\kappa = 0$  the action (1)

is equivalent to the Eddington-Born-Infeld (EBI) theory proposed in [18] as a theory for dark matter and dark energy. When one generalizes to the case  $\kappa \neq 0$ , it is a natural question to ask whether or not the dark matter/dark energy interpretation still holds. The answer is in the affirmative. The metric  $q_{\mu\nu}$  can behave both as matter or as dark energy, and there exist solutions interpolating between them. We present two types of de Sitter vacua and study their stability under a certain set of perturbations. One is the well known solution in which the metrics are proportional. In the other case, which has received less attention, the de Sitter line elements of the two metrics are not proportional. These type of backgrounds were pointed out in different contexts (e.g., in [19] for the flat case, and in [20] in static coordinates). We also display what are the conditions on the couplings that determine whether the Universe evolving from a matter era ends up in the proportional or not proportional vacuum. Finally, we analyze tensor fluctuations on the de Sitter backgrounds. For the proportional case the equations can be decoupled and a condition on the couplings ensuring absence of tachyons is displayed.

The large scale structure of the action (1) with  $\kappa = 0$ , called EBI theory, has recently been studied in Ref. [21]. In that article, it was shown that the EBI theory has a phase for which the Friedmann background evolution, growth of inhomogeneities and Cosmic Microwave Background (CMB) angular power spectrum are indistinguishable from those predicted by  $\Lambda$ CDM. These results provide extra support for these theories as candidates for dark matter/dark energy. We shall come back to this point at the end.

Theories interpolating between dark matter and dark energy are not new. Examples are the Chaplygin gas[22, 23, 24] and the rolling tachyon[25, 26, 27]. For the Chaplygin gas, observational consistency of this interpolation has been challenged in Ref.[28].

We start by analyzing the relationship between the action (1) when  $\kappa = 0$  and the EBI theory written in [18]. This is straightforward. In what follows it is convenient

to refer all lengths to  $\ell$ . We thus define two new dimensionless couplings  $\alpha$  and  $\alpha_0$  by

$$\Lambda = \frac{\alpha_0}{\ell^2}, \quad \lambda = \frac{\alpha}{\ell^2}. \quad (2)$$

The first step is to write the action (1) in Palatini form with a connection  $C^\mu_{\alpha\beta}$  associated to the metric  $q_{\mu\nu}$ . Varying with respect to  $q_{\mu\nu}$ , the equations can be algebraically solved for this field,

$$q_{\mu\nu} = \frac{1}{\lambda} \left( K_{\mu\nu} - \frac{1}{\ell^2} g_{\mu\nu} \right) \quad (3)$$

and therefore the solution can be replaced back in the action. The resulting action depends on  $g_{\mu\nu}$  and the connection  $C^\mu_{\alpha\beta}$  and is precisely the EBI action,

$$I_{\text{EBI}}[g, C] = \frac{1}{16\pi G} \int d^4x \left\{ \sqrt{-g} (R - 2\Lambda) + \frac{2}{\alpha\ell^2} \sqrt{-\det(g_{\mu\nu} - \ell^2 K_{\mu\nu})} \right\}. \quad (4)$$

Note that the action (1), for  $\kappa = 0$ , in its Palatini form, is a *parent action* in the sense that one may either eliminate  $q_{\mu\nu}$  to obtain EBI, or eliminate the connections to get (1) in terms of the metrics only.

Born-Infeld type actions have appeared repeatedly for many years in many different contexts. The action (4) is particularly close to the one discussed in [29] although different in interpretation. Another class of Born-Infeld theories are tachyonic fields[25, 26], which are another candidate for dark matter and dark Energy[30]. This field is described by the following effective action,

$$I_t = - \int d^4x V(\phi) \sqrt{-\det(g_{\mu\nu} + \partial_\mu \phi \partial_\nu \phi)} + I_{EH}(g_{\mu\nu}), \quad (5)$$

where  $I_{EH}(g_{\mu\nu})$  is the Einstein-Hilbert action including a cosmological constant and  $V(\phi)$  is an effective potential, which, in open string theory is[31]

$$V(\phi) = \frac{V_0}{\cosh(a\phi)}. \quad (6)$$

Just as the square root in the EBI theory (4) can be transmuted into a standard kinetic term by introducing  $q_{\mu\nu}$ , a similar manipulation holds for (5). Consider the following action,

$$I_p = V_0 \int \sqrt{-q} \left( -\frac{1}{2} q^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - U(\phi) - \frac{1}{2} q^{\alpha\beta} g_{\alpha\beta} \right). \quad (7)$$

This action can be seen as Polyakov's version of (5), except that the metric field is  $q_{\mu\nu}$ . This is seen by varying with respect to  $q^{\alpha\beta}$ . We obtain an equation which allows us to algebraically solve  $q_{\alpha\beta}$  in terms of  $\phi$ , therefore, we may put it back in the action (7). We get

$$I'_p = \int d^4x \frac{V_0}{U(\phi)} \sqrt{-\det(g_{\mu\nu} + \partial_\mu \phi \partial_\nu \phi)}, \quad (8)$$

which is precisely the tachyon action (5) action when  $U(\phi) = -V_0/V(\phi)$ .

Considering the form of action (7), it is suggestive to add a kinetic term to the auxiliary metric field  $q_{\mu\nu}$ . The obvious choice is to also add a Einstein Hilbert term with a cosmological constant. If we do so, we obtain precisely the first line of (1) plus  $I_p$  in (7). This is EBI action in bi-gravity form plus a scalar field minimally coupled to the metric  $q_{\mu\nu}$ . The variation of this action with respect to  $q_{\mu\nu}$  gives, again, an equation that may be solved algebraically for the  $q$ -metric. Inserting this back in the action, and redefining  $V_0$ , we obtain the following generalization of (4),

$$I = \frac{1}{16\pi G} \int \left\{ \sqrt{-g} \left( R - \frac{2\alpha_0}{\ell^2} \right) + \frac{4}{\ell^2(U + 2\alpha)} \sqrt{-\det(g_{\mu\nu} - \ell^2 K_{\mu\nu} + \partial_\mu \phi \partial_\nu \phi)} \right\}. \quad (9)$$

which becomes EBI when  $U = \phi = 0$ .

Our second goal is to study the cosmological properties of the bi-gravity system described by (1). Assuming that both metrics are homogeneous and isotropic flat FRW,

$$ds_g^2 = -dt^2 + a^2 d\vec{x}^2, \quad ds_q^2 = -X^2 dt^2 + Y^2 d\vec{x}^2. \quad (10)$$

For a given set of couplings  $\alpha, \alpha_0, \kappa$  there exists more than one de Sitter vacua. We distinguish two cases: proportional vacuums (PV) if both metrics are proportional and non-proportional vacuums (NPV). In the proportional case the functions  $a, X, Y$  are given by

$$a = e^{\frac{H}{\ell}t}, \quad X^2 = \frac{1 - (\alpha_0 - 6\kappa)}{1 - \alpha} \quad Y = aX \quad (11)$$

with

$$H^2 = \frac{1 - (\alpha_0 - 6\kappa)\alpha}{3(1 - \alpha)}. \quad (12)$$

The only condition for the existence of this solution is the positivity of the constants  $H^2$  and  $X^2$  above. Note that if  $\alpha_0 - 6\kappa = \alpha^{-1}$ , then this vacuum becomes Minkowski. Also note that for  $\alpha$  close enough to 1, or  $\kappa$  sufficiently large, the de Sitter acceleration can be made arbitrarily large, even if the cosmological constant,  $\alpha_0/\ell^2$ , vanishes. Conversely, even for big values of  $\alpha_0$  we may fine-tune the couplings in order to obtain arbitrarily small acceleration. This is an attractive feature in the context of the problem of the cosmological constant.

A second class of de Sitter vacua with non-proportional metrics also exists (NPV). Let  $Y = aXA$ , where  $A$  is a constant (which is 1 for the previous case). In this case  $a = e^{\frac{H}{\ell}t}$  where  $A, X$  and  $H$  are constants determined by the equations:

$$\begin{aligned} \kappa &= \frac{X^2 A^2}{4} & \alpha_0 &= 3H^2 + \frac{X^2 A^3}{2} \\ \alpha &= -\frac{3}{4X^2 A^2} + \frac{3H^2}{X^2} - \frac{1}{4X^2} \end{aligned} \quad (13)$$

To find the metric parameters  $H$ ,  $A$ ,  $X$  one needs to solve a third order algebraic equation. This means that, in general, we may expect three different NPV for a given set of couplings. In [19], these kind of solutions are also discussed. In that case, however, the cosmological constants are adjusted to have flat backgrounds, so the mixing term used here, called  $\mathcal{V}_1$  in that reference, gives rise only to the proportional vacua.

We may ask now if the above vacua are stable. Consider perturbations of the form

$$\begin{aligned} a(t) &= a_0(t) + \epsilon a_1(t), \\ X(t) &= X_0(t) + \epsilon X_1(t), \\ Y(t) &= Y_0(t) + \epsilon Y_1(t), \end{aligned}$$

where the subscript  $_0$  indicates the background solutions found above. We expand the equations to linear order in  $\epsilon$  and look for the conditions on the couplings such that the perturbations do not grow in time. These conditions are best expressed with a picture. In Fig. 1, we show the regions of stability for the PV and the NPV with  $\alpha_0 = 1/2$  and  $\alpha = 0.9$  respectively.

The vertical axis represent the value of  $\alpha$ . Note that there are regions where both solutions are stable. Those are the regions where we have degenerated vacuum. We do not know, however, if one of them turns out to be metastable.

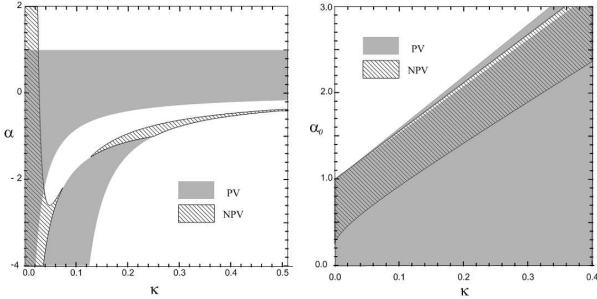


FIG. 1: Stability regions for the proportional vacuum solution (PV) and the non-proportional ones (NPV). In the left the value of  $\alpha_0 = 0.5$  is fixed. In the right  $\alpha = 0.9$  is fixed.

Now we proceed to show, by studying the background cosmological evolution, that this theory encompass a dark matter candidate, even for  $\kappa \neq 0$ . To this end, the equations of motion coming from (1) are evaluated with homogeneous and isotropic metrics. We assume that both metrics are not singular at the same time [37], which implies that, when the scale factor  $a(t)$  [with  $a(0) = 0$ ] is sufficiently close to zero,

$$X(t) \approx X_0 + X_1 t, \quad Y(t) \approx Y_0 + Y_1 t, \quad (14)$$

where  $X_0, Y_0, X_1, Y_1$  are constants. Inserting this into the equations of motion, with  $q$  and  $g$  metrics given by (10), one finds  $Y_1$  and  $X_1$  as a function of  $X_0, Y_0, \alpha$ , and

$\ell$ . The Friedmann equation for  $a(t)$  in the limit of  $a(t)$  sufficiently small reads,

$$3H^2 \approx 8\pi G\rho + \frac{Y_0^3}{\ell^2 X_0} \frac{1}{a^3}. \quad (15)$$

where  $\rho$  is the energy density of other conventional fluids (e.g. radiation or baryons). Therefore, we see that the  $q$ -metric plays the role of an additional dust-like matter, irrespective of the presence of other conventional fluids like radiation or baryons. Adjusting the constant  $\frac{Y_0^3}{\ell^2 X_0}$  one can have any desired amount of “dark matter”. In particular choosing  $\frac{Y_0^3}{\ell^2 X_0} = 3.34 \times 10^{-7} w_c Mpc^{-2}$  where  $w_c \sim 0.09 - 0.12$  we get the right amount of dark matter as required by cosmological observations. Note that the  $\kappa$  constant has no role at the above regime, and so both the bi-gravity action (1) and the EBI model describes the same physics when  $a(t) \ll 1$ .

Our final task is to study gravitons propagating on the de Sitter vacua. In the following we use conformal time. We start by perturbing the FRW metrics as

$$ds_g^2 = a^2 [-d\tau^2 + (\gamma_{ij} + h_{ij})dx^i dx^j] \quad (16)$$

and

$$ds_q^2 = -a^2 X^2 d\tau^2 + Y^2 (\gamma_{ij} + \chi_{ij})dx^i dx^j \quad (17)$$

where  $h_{ij}$  is the tensor mode perturbation of the  $g$ -metric and  $\chi_{ij}$  the tensor mode perturbation of the  $q$ -metric. The tensor modes are transverse and traceless.

From now on we drop the indices on  $h_{ij}$  and  $\chi_{ij}$  since no confusion arises. We find that the field equations for  $h$  and  $\chi$  are

$$\begin{aligned} \ddot{h} + 2\frac{\dot{a}}{a}\dot{h} - \vec{\nabla}^2 h = \\ -\frac{2a^2}{\ell^2 \sqrt{-w_0}} [X^2 - 2(1 - w_0)\kappa] (h - \chi) \end{aligned} \quad (18)$$

where  $w_0 = -\frac{a^2 X^2}{Y^2}$ , and

$$\begin{aligned} \ddot{\chi} + \left( 3\frac{\dot{Y}}{Y} - \frac{\dot{a}}{a} - \frac{\dot{X}}{X} \right) \dot{\chi} + w_0 \vec{\nabla}^2 \chi = \\ -\frac{2a^2 w_0}{\ell^2 X^2} [X^2 - 2(1 - w_0)\kappa] (h - \chi) \end{aligned} \quad (19)$$

respectively. We now adopt the above equations to the special case of de Sitter vacua. For the proportional de Sitter vacuum described above with  $\frac{\dot{X}}{X} = 0$ ,  $\frac{\dot{Y}}{Y} = \frac{\dot{a}}{a}$  and  $w_0 = -1$  these two equations can be collected in matrix form as

$$\left[ \frac{\partial^2}{\partial \tau^2} + 2\frac{\dot{a}}{a} \frac{\partial}{\partial \tau} - \vec{\nabla}^2 + a^2 \mathcal{M}^2 \right] \begin{pmatrix} h \\ \chi \end{pmatrix} = 0 \quad (20)$$

where the mass matrix  $\mathcal{M}^2$  is

$$\mathcal{M}^2 = \frac{2}{\ell^2} (X^2 - 4\kappa) \begin{pmatrix} 1 & -1 \\ -\frac{1}{X^2} & \frac{1}{X^2} \end{pmatrix} \quad (21)$$

One of the eigenvalues is clearly zero while the other one is

$$m^2 = \frac{2}{\ell^2} (X^2 - 4\kappa) \left( 1 + \frac{1}{X^2} \right) \quad (22)$$

Thus in order for the theory not to contain spin-2 tachyons, we must have  $X^2 > 4\kappa$  which translates to

$$\frac{1 - \alpha_0 + 2(1 + 2\alpha)\kappa}{1 - \alpha} > 0 \quad (23)$$

If these conditions are fulfilled, this theory describes a massless and massive graviton. In particular, they imply stability for the EBI vacuum tensorial modes, which were first studied in Ref.[32].

It is important to mention that gravitons are not the only physical excitations; vector and scalars modes may also propagate. The reason is that the action has two metrics but only the diagonal subgroup of diffeomorphisms leaves the action invariant. This means that the scalar and vector modes of only one of the metrics can be set to zero by a gauge symmetry. This raises the issue of the stability of the theory which should be analyzed along the lines of Ref.[33, 34]. It was shown in those references that cosmological massive gravitons are stable if the mass of the graviton obeys  $m^2 > 2\Lambda/3$ . We expect similar results to hold in our case.

To conclude, we have studied in this note several cosmological aspects of bigravity actions of the form (1).

Most importantly we have shown that generically these actions contain a phase at early times where the second metric behaves as dark matter. The equations also admit a de-Sitter background which is an attractor if some conditions on the couplings are fulfilled. This implies a transition between the “matter” and “de Sitter” phases. For the case  $\kappa = 0$  this transition has been explored in detailed in [21] and shown to be problematic at the level of fluctuations and the calculation of CMB spectra. However, as shown in [21], one can choose initial conditions and couplings such that the metric  $q_{\mu\nu}$  is locked into its matter phase up until today. In this case, bigravity predicts a CMB spectrum which is indistinguishable from standard particulate dark matter. The calculations for  $\kappa \neq 0$  are far more complicated and we shall consider them in a separate publication.

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  - [37] This ansatz is in close relation to the proposal of having meaningful physics even when  $g_{\mu\nu} = 0$  explored in [18, 35].